Abstract booklet for “Nonlinear Waves in Sicily”

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Nonlinear waves photonic lattices

In the study of photonic lattices with simple periodic potentials, discrete and continuous NLS equations arise. In non-simple periodic, hexagonal or honeycomb lattices, new discrete Dirac-like systems and their continuous analogs can be derived. They are novel nonlinear systems and have interesting properties. Honeycomb lattices also occur in the material graphene; the optical case is termed photonic graphene.

Nalan Antar

Istanbul Technical University
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Lattice solitons in $\mathcal{PT}$ symmetric lattices with defects

In this talk, I will discuss the existence of lattice solitons supported by cubic-saturable nonlinerarity , in the framework of nonlinear Schrödinger equation with PT symmetric lattices with defects by using the pseudo-spectral renormalization method. The idea behind the spectral renormalization method is to transform the governing equation into Fourier space and find a nonlinear nonlocal integral equation coupled to an algebraic equation and determine a convergence factor based upon the degree (homogeneity) of a single nonlinear term. The convergence factor can not found explicitly from the governing equation for saturable case by the use of the spectral renormalization method . In order to find the convergence factor has to be used the root finding code such as the Newton method but if we use the pseudo-spectral renormalization method the convergence parameter is found from the governing equation explicitly. The pseudo-spectral renormalization method can efficiently be applied to a large class of problems including higher order nonlinear terms with different homogeneities. In first part of my talk, I will present the pseudo-spectral renormalization method for cubic-saturable nonlinearity. I report the existence and stability of the lattice solitons in parity-time $\mathcal{PT}$ symmetric lattices with defects in focusing/defocusing cubic-saturable media.

The propagation of light beams along the z- axis of the medium composed from alternating domains with cubic and saturable nonlinearities is described by nonlinear Schrödinger equation (NLSE) with the external potential

$$i\frac{\partial u}{\partial z}+\frac{\partial^2 u}{\partial x^2}+\frac{\partial^2 u}{\partial y^2}+\sigma\frac{|u|^2 u}{1+\alpha |u|^2}+V(x,y)u=0$$

Saturation parameter $\alpha = 0$ and $\alpha = 1$ correspond to the cubic domain and the saturable domain respectively. I will consider two different configuration. I will investigate the linear and nonlinear stability properties of the lattice solitons in PT symmetric lattices in two different configuration. It is found that for $\mathcal{PT}$ symmetric lattices the solitons suffer collapse in the cubic domain. I will discuss the stability properties (Nonlinear and Linear Spectrum) of the lattice solitons in the saturable domain with PT symmetric lattice with defects.

Ilkay Bakirtaş

Istanbul Technical University
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Fundamental and Multi-humped Solitons in nonlocal nonlinear Schrödinger type equations

In this talk, fundamental and multi-humped (dipole and 4-vortex) solitons in nonlocal nonlinear Schrödinger (NLS) systems with an external potential (lattice) will be presented. Such systems can be written in the following non-dimensional form,

$$iu_z + \frac{1}{2}(u_{xx} + u_{yy}) + |u|^2 u - \rho\phi u - V(x,y)u = 0$$

$$\phi_{xx} + \nu\phi_{yy} = (|u|^2)_{xx}$$

where u is the normalized amplitude of the envelope of the optical beam and ϕ is the normalized static field, ρ is the coupling constant which comes from the combined optical rectification - electro optic effect, ν is the asymmetry parameter comes from the anisotropy of the material and $V(x,y)$ is an external optical potential.
In this study, the fundamental and multi-humped solitons of NLSM system with a periodic lattice are computed by using the spectral renormalization method. In current literature, in (2+1)D, solitons of NLSM systems (without external potential) and solitons of NLS equation (with or without external potentials) are shown to suffer from wave collapse. Therefore, understanding/arresting collapse in such systems is an important challenge.
In the second part of my talk, I will discuss the nonlinear stability properties of the optical modes (solitons) of NLSM systems for a parameter space.

Gino Biondini

SUNY Buffalo
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The integrable nature of modulational instability

The modulational instability (MI) [known as Benjamin-Feir instability in water waves], is one of the most widespread phenomena in nonlinear science. In many cases, the underlying dynamics of systems subject to MI is governed by the focusing nonlinear Schrodinger (NLS) equation. The initial stage of MI can thus be described by linearizing the NLS equation around a constant background. Once the perturbations have become O(1), however, the linearization ceases to be valid. This regime is called the nonlinear stage of MI, and a full characterization of it has remained an open problem for over forty years. On the other hand, the NLS equation is a completely integrable infinite-dimensional Hamiltonian system, and the initial-value problem is therefore amenable to solution via the inverse scattering transform (IST). In this talk I will present the recently-developed IST for the focusing NLS equation with non-zero boundary conditions at infinity and I will describe how it can be used to study the nonlinear stage of MI.

Sonia Boscolo

Aston University
Birmingham, UK

Temporal and Spectral Nonlinear Pulse Shaping in Normally Dispersive Optical Fibers

The combination of the third-order optical nonlinearity with chromatic dispersion in optical fibers offers an extremely rich variety of possibilities for tailoring the temporal and spectral content of a light signal, depending on the regime of dispersion that is used. Because the nonlinear dynamics of pulses propagating in fibers with normal group-velocity dispersion is generally sensitive to the initial pulse condition, it is possible to nonlinearly shape the propagating pulses through control of the initial pulse temporal intensity and/or phase profile. Here, we review recent progress on the use of third-order nonlinear processes in normally dispersive fibers for pulse shaping in the temporal and spectral domains. Various examples of practical significance will be discussed, spanning fields from the generation of specialized temporal waveforms to the generation of stable continua with high power spectral density, and to spectral compression of the pulses.

Francesco Demontis

Università di Cagliari
Cagliari, Italy

Exact Solution and Vortex Filament for the Hirota Equation

By using the Inverse Scattering Transform we construct an explicit soliton solution formula for the Hirota equation. The formula obtained allows one to get, as a particular cases, the N-soliton solution, the breather solution and, most relevantly, a new class of solutions called multipole soliton solutions. By adapting the Sym-Pohlmeyer reconstruction formula to the Hirota equation, we use these exact solutions to study the motion of a vortex filament in an incompressible Euler fluid with nonzero axial velocity. This talk is based on a joint work with G. Ortenzi (University of Bergamo) and C. van der Mee (University of Cagliari).

Theodoros Horikis

University of Ioannina
Ioannina, Greece

Monsters of the deep: Rogue Waves

It is common for mid-ocean storm waves to reach seven meters in height, and in extreme conditions such waves can reach heights of fifteen meters. However, for centuries maritime folklore told of the existence of vastly more massive waves that could appear without warning in mid-ocean, against the prevailing current and wave direction, and often in perfectly clear weather. These waves are called rogue waves. A rogue wave is a highly localized phenomenon both in space and duration, most frequently occurring far out at sea. Historically oceanographers have discounted these reports as tall tales, i.e. the embellished stories of mariners with too much time at sea. But in the last years scientists have discovered strong evidence indicating that such massive rogue waves do exist and while the phenomenon has become the subject of recent scientific study, their origin still remains a mystery of the deep. The generation mechanisms of these waves will be the focus of this talk.

Boaz Ilan

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Onset of transverse instabilities of confined dark solitons

We investigate existence, stability and dynamics of dark soliton solutions of the nonlinear Schrödinger / Gross-Pitaevskii (NLS/GP) equation that are confined to propagate in 2D channels and 3D pipes. These solitons are unstable to small transverse perturbations, unless they are sufficiently confined along the transverse direction. The onset of this transverse instability is characterized in terms of a spectral bifurcation of the discrete spectrum of the linearized equation. The bound states are computed using a spectrally accurate fixed-point iterative scheme. The critical confinement width at which this bifurcation occurs is found to agree with the onset of a transverse snake instability observed in direct numerical computations of the NLS/GP equation. These results have application to the control of ultracold matter waves and intense laser beams. Joint work with Mark Hoefer and Nicholas Lowman.

Sara Lombardo

Northumbria University
Newcastle upon Tyne, UK

Rogue waves of resonant-interactions models

Rogue waves can be modelled by rational solutions of nonlinear dispersive equations. While the Nonlinear Schroedinger equation (NLS) has played a central role as a universal model for rogue wave solutions, moving beyond the standard focusing NLS description in order to model more general and important classes of physical systems is both relevant and necessary. Since numerous physical phenomena require modelling waves with two or more components in order to account for different modes, frequencies, or polarizations, important progress has been recently obtained by extending the search for rogue wave solutions to coupled-wave systems. When compared to scalar systems, vector systems may allow for energy transfer between their additional degrees of freedom, which potentially yields rich and significant new families of vector rogue wave solutions.

In this talk I will report on the search for rogue waves in resonant-interaction models of applicative interest in many fields, focussing on integrable models which describe the resonant interaction of two or more waves in 1+1 dimensions. In particular, I will consider a system of three coupled wave equations, which includes as special cases, the vector Nonlinear Schroedinger equation (or Manakov System) with both self- and cross-focusing/defocusing interaction terms, and the equations describing the resonant interaction of three waves. The Darboux-Dressing transformation is applied under the condition that the solutions have rational, or mixed rational-exponential, dependence on coordinates, leading to an algebraic construction which relies on nilpotent matrices and their Jordan form and it allows for a systematic search of all bounded rational (mixed rational-exponential) solutions.

Luigi Martina

Università del Salento
Lecce, Italy

Waves in Skyrme-Faddeev Model

Special reductions of the Skyrme-Faddeev model admit exact analytical solutions, which may describe magnetic domain wall solutions when singularities appear. Differently, always regular periodic nonlinear waves can be found.

They may degenerate into linear spin waves or solitonic structures. Here both classes of solutions are derived and discussed and the existence of integrable subsectors of the model is addressed.

Kenichi Maruno

Waseda University
Tokyo, Japan

Self-adaptive moving mesh schemes for short pulse type equations and their Lax pairs

I will talk about Integrable self-adaptive moving mesh schemes for short pulse type equations (the short pulse equation, the coupled short pulse equation, and the complex short pulse equation). I propose two systematic methods to create self-adaptive moving mesh schemes, one is based on bilinear equations and another is based on Lax pairs. Self-adaptive moving mesh schemes consist of two semi-discrete equations in which the time is continuous and the space is discrete. In self-adaptive moving mesh schemes, one of two equations is an evolution equation of mesh intervals which is deeply related to a discrete analogue of a reciprocal (hodograph) transformation. An evolution equations of mesh intervals is a discrete analogue of a conservation law of an original equation, and a set of mesh intervals corresponds to a conserved density which play an important role in generation of adaptive moving mesh. Lax pairs of self-adaptive moving mesh schemes for short pulse type equations are obtained by discretization of Lax pairs of short pulse type equations, thus the existence of Lax pairs guarantees the integrability of self-adaptive moving mesh schemes for short pulse type equations. It is also shown that self-adaptive moving mesh schemes for short pulse type equations provide good numerical results by using standard time-marching methods such as the improved Euler's method.

Antonio Moro

Northumbria University
Newcastle upon Tyne, UK

Shock dynamics of phase diagrams

A thermodynamic phase transition denotes a drastic change of state of a physical system due to a continuous change of thermodynamic variables, as for instance pressure and temperature. The classical van der Waals equation of state is the simplest model that predicts the occurrence of a critical point associated with the gasliquid phase transition. Nevertheless, below the critical temperature theoretical predictions of the van der Waals theory significantly depart from the observed physical behaviour. We develop a novel approach to classical thermodynamics based on the solution of Maxwell relations for a generalised family of nonlocal entropy functions. This theory provides an exact mathematical description of discontinuities of the order parameter within the phase transition region, it explains the universal form of the equations of state and the occurrence of triple points in terms of the dynamics of nonlinear shock wave fronts.

Ziad Musslimani

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Integrable Nonlocal Nonlinear Schrödinger Equation

A new integrable nonlocal nonlinear Schrödinger equation is introduced. It possesses a Lax pair and an infinite number of conservation laws and is PT symmetric. The inverse scattering transform and scattering data with suitable symmetries are discussed. A method to find pure soliton solutions is given. An explicit breathing one soliton solution is found. Key properties are discussed and contrasted with the classical nonlinear Schrödinger equation. This is a joint work with Mark J. Ablowitz from the University of Colorado at Boulder.

Vladimir Novikov

Loughborough University
Loughborough, UK

Integrable equations in 3D: deformations of dispersionless limits

Classification of integrable systems remains as a topic of active research from the beginning of soliton theory. Numerous classification results are obtained in 1 + 1 dimensions by means of the symmetry approach. Although the symmetry approach is also applicable to 2 + 1-dimensional systems, one encounters additional difficulties due to the appearance of nonlocal variables. There are several techniques to tackle the problem (e.g. the perturbative symmetry approach). In the perturbative symmetry approach one starts with a linear equation having a degenerate dispersion law and reconstructs the allowed nonlinearity.

In this talk we present a novel perturbative approach to the classification problem. Based on the method hydrodynamic reductions, we first classify integrable quasilinear systems which may potentially occur as dispersionless limits of integrable 2 + 1-dimensional soliton equations. Subsequently we construct dispersive deformations preserving integrability deforming the hyrdrodynamic reductions by dispersive deformations and requiring that all hydrodynamic reductions of the dispersionless limit will be inherited by the corresponding dispersive counterpart. The method also allows to effectively reconstruct Lax representations of the deformed systems. We present various classification results obtained in the frame of the new approach, e.g. the classification of scalar 2+1-dimensional equations generalizing KP, BKP/CKP, the classification of Davey-Stewartson type systems as well as various classifications of 2 + 1-dimensional differential-difference equations.

The talk is based on joint work with E. Ferapontov, A. Moro, B. Huard and I. Roustemoglou.

Barbara Prinari

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Università del Salento
Lecce - Italy

Inverse Scattering Transform for nonlinear Schrödinger equations with non-zero boundary conditions

We will discuss some recent results on the Inverse Scattering Transform (IST) for the focusing and defocusing nonlinear Schrödinger equations: $iq_t = q_{xx} + 2\sigma|q|^2 q$, $\sigma = \pm 1$, with nonzero boundary values $q_\pm(t) \equiv |q_\pm| e^{i\theta_\pm(t)}$

as $x \to \pm\infty$. In both dispersion regimes, the direct problem is shown to be well-posed for potentials $q(x,t)$ such that $q(x,t) - q_\pm(t) \in L^{1,1}(\mathbb{R}^\pm)$ with respect to x for all $t \geq 0$, for which analyticity properties of eigenfunctions and scattering data can be rigorously established. The inverse scattering problem is formulated both via (left and right) Marchenko integral equations, and as a Riemann-Hilbert problem on a single sheet of the scattering variables $\lambda_\pm = \sqrt{k^2 + \sigma|q_\pm|^2}$, $k \in \mathbb{C}$ being the usual scattering parameter in the IST.

Besides setting up the stage for a rigorous study of the long-time asymtptotics of fairly general NLS solutions, the IST formulation enabled us to further investigate the spectrum of the associated scattering operators. Specifically, for the defocusing NLS equation with nonzero boundary conditions having $|q_+| = |q_-|$, by considering a specific kind of piecewise constant initial condition we were able to clarify two issues, concerning: (i) the (non)existence of an area theorem relating the presence/absence of discrete eigenvalues to an appropriate measure of the initial condition; and (ii) the existence of a contribution to the asymptotic phase difference of the potential from the continuous spectrum.

This talk is based on joint work with Gino Biondini and Emily Fagerstrom (SUNY Buffalo), Francesco Demontis and Cornelis van der Mee (Università di Cagliari), and Federica Vitale (Università del Salento).

Cornelia Schiebold

Mid Sweden University
Sundsvall, Sweden

NLS-systems and the operator method

Our point of departure is an operator theoretic approach to soliton equations, which is inspired by work of Marchenkov and enables us to Banach geometry in the study of solution families. In the talk we give an outline how the operator method can be applied to NLS-systems and discuss some applications such as the asymptotic description of multiple pole solutions, the construction of solutions to matrix equations and countable nonlinear superposition.

Matteo Sommacal

Northumbria University
Newcastle upon Tyne, UK

One- and two-dimensional magnetic-droplet solitons

Recent results on propagating, solitary magnetic wave solutions of the Landau-Lifshitz (LL) equation with uniaxial, easy-axis anisotropy in thin (one- and two-dimensional) magnetic films will be illustrated. These localized, nontopological wave structures, parametrized by their precessional frequency and propagation speed, extend the stationary, coherently precessing “magnon droplet” to the moving frame, a non-trivial generalization due to the lack of Galilean invariance. Propagating droplets move on a spin wave background with a nonlinear droplet dispersion relation that yields a limited range of allowable droplet speeds and frequencies. The droplet is found to propagate as a Nonlinear Schrödinger bright soliton in the weakly nonlinear regime. Using spin transfer torque underneath a nanocontact on a magnetic thin film with perpendicular magnetic anisotropy (PMA), the generation of dissipative magnetic droplet solitons was announced this year for the first time, following its theoretical prediction. Rich dynamical properties (including droplet oscillatory motion, droplet spinning, and droplet breather states) have been experimentally observed and reported. After reviewing the conservative magnetic droplet, some properties of the soliton in a lossy medium will be discussed. In particular, it will be shown that the propagation of the dissipative droplet can be accelerated and controlled by means of an external magnetic field. Soliton perturbation theory corroborated by two-dimensional micromagnetic simulations predicts several intriguing physical effects, including the acceleration of a stationary soliton by a magnetic field gradient, the stabilization of a stationary droplet by a uniform control field in the absence of spin torque, and the ability to control the solitons speed by use of a time-varying, spatially uniform external field. Soliton propagation distances approach 10um in low-loss media, suggesting that droplet solitons could be viable information carriers in future spintronic applications, analogous to optical solitons in fiber optic communications. Finally, if the time will allow, some novel results, obtained in collaboration with C. van der Mee, F. Demontis and S. Lombardo, concerning the inverse scattering theory for the eigenvalue problem associated with the one-dimensional isotropic LL equation, will be presented.

Nikola Stoilov

University of Göttingen
Germany

'Riemann Equations' in bidifferential calculus

In the algebraic framework of bidifferential calculus, we consider equations that formally resemble a (matrix) Riemann equation. De- pending on the choice of first-order bidifferential calculus, these 'Riemann equations' can represent a variety of equations, and several examples are discussed. A recent (non-isospectral) binary Darboux transformation result in bidifferential calculus is specialised to generate solutions of the 'Riemann equations'. In particular, we obtain multi-kink solutions of semi-discrete and fully discrete versions of the Rie- mann equation. These equations and the solution-generating method extend to hierarchies. Furthermore, if the bidifferential calculus does extend to second order, solutions of a system of 'Riemann equations' are also solutions of the equation that arises as an integrability condition. Depending on the choice of bidifferential calculus, the latter can represent a number of prominent integrable equations, like self-dual Yang-Mills, (matrix versions of) two-dimensional Toda lattice, a variant of Hirota's bilinear difference equation, KP and Davey-Stewartson equations.
Joint work with O. Chvartatskyi, A. Dimakis and F. Müller-Hoissen

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Stefano Trillo

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Resonant Radiation shed by Optical Dispersive Shock Waves

Dispersive shock waves (DSWs) have been the focus of intense research efforts in the last years. In this respect optics offers the unsurpassed ability to observe DSWs in spatial and temporal settings which can be controlled with extremely high accuracy. Usually DSWs develop in the regime where the dispersive effects are weak compared with those due to the nonlinearity. Common models employed in optics, such as the semiclassical (weakly dispersing) nonlinear Schrödinger (NLS) equation, usually accounts for a second-order dispersion of the normal type (defocusing NLS). However, when the latter is small, higher-order corrections become important. In this talk we show that these corrections induces the DSWs to radiate at characteristic frequencies that can be predicted by means of a perturbative analysis. We consider both conservative and dissipative (driven-damped) models. The former case applies to the free-propagation along an optical fiber, described in terms of the generalized NLS equation with dispersion at all orders [1,2]

$$\begin{gathered} i\varepsilon\partial_z\psi + d(\partial_t)\psi + |\psi|^2\psi = 0, \\ d(\partial_t) = \sum_n \frac{\beta_n}{n!}\varepsilon^n\,(i\partial_t)^n = -\beta_2\frac{\varepsilon^2}{2}\partial_t^2 - i\beta_3\frac{\varepsilon^3}{6}\partial_t^3 + \beta_4\frac{\varepsilon^4}{24}\partial_t^4 + \ldots \end{gathered} \tag{1}$$

where $\beta_2 = \pm 1$ and $\beta_3, \beta_4, \ldots$ are free coefficients that fix the impact of higher-order dispersion. This model is important in order to contrast the radiative processes driven by bright solitons ($\beta_2 = -1$, focusing NLS) and DSWs ($\beta_2 = 1$, defocusing NLS), respectively. In the latter case we find the frequency of the linear waves to be strongly

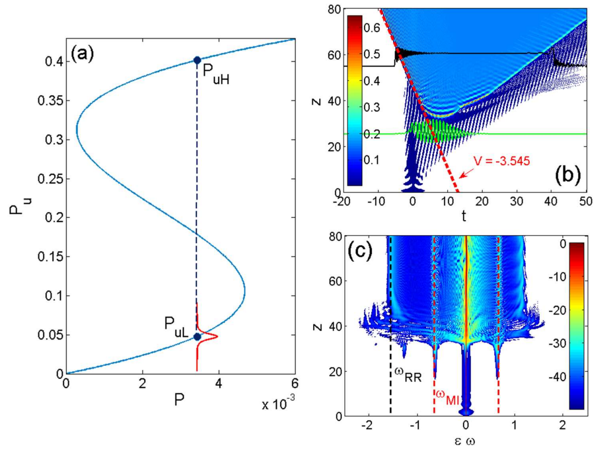

Figure 1: Radiating DSWs ruled by Eq. (2) with $\varepsilon = 0.1$, $\beta_2 = 1$, and $\beta_3 = 2$: (a) Stationary bistable response $P_u = |u|^2$ vs. driving P, with superimposed pulsed biased excitation (red curve); (b) Temporal evolution. The red dashed line gives the slope corresponding to the velocity of the radiating shock; (c) Spectral evolution showing spectral lines due to resonant radiation (ω_{RR}) and modulation instability (ω_{MI}).

affected by the shock velocity. Moreover resonant radiation is emitted from DSWs according to a host of different physical situations depending on the values of the parameters β_3 and β_4. In the limit of perturbative third-order dispersion our theory explains the origin of the generalized dispersive radiation observed as a spectral peak appearing across the zero dispersion point when pumping a fiber with pulses in the normal group velocity dispersion regime [3].

We also discuss the case of fiber ring resonators which are described by a driven-damped NLS model (known also as Lugiato-Lefever model)

$$i\epsilon\partial_z u + d(i\partial_t)u + |u|^2 u = (\delta - i\alpha)\, u + i\sqrt{P}, \tag{2}$$

where the terms on the RHS accounts for cavity detuning (δ), resonator losses (α) and external driving with power $P(t)$. The salient feature of Eq. (2) is the fact that it introduces bistability of the steady-state solutions. Such bistability favors the formation of shock waves which can be considered of the mixed dispersive-dissipative type, with dominant dispersive nature in the limit of weak losses α. We discuss how to dynamically excite such DSWs and predict the radiated frequencies. The dynamics is complicated by the occurrence of modulational instability (MI) which also favor the shock under appropriate conditions. An example of the simulated evolution is shown in Fig. 1. The cavity is driven by an activating pulse on top of a continuous wave bias. Shock waves that connect the lower branch [state with power $|u|^2 = P_{uL}$ in Fig. 1(a)] and upper branch [state with power $|u|^2 = P_{uH}$ in Fig. 1(b)] of the bistable response are formed via MI. The process ends up showing main spectral peaks associated with both MI and resonant radiation shed by the shock front.

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